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A Possible Construction of the Quantum Field Theory

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Abstract

A method of construction of quantum field theory as a limit of some approximate theories is proposed. The way to find the limit is indicated and discussed in more detail for the free field. Suggestions for interacting fields are also given.

1. *Introduction*

In recent years J. Glimm and A. M. Jaffe have proved the existence of a non-trivial model in two-dimensional space-time (Glimm & Jaffe, 1968, 1970). Their proof consists in achieving the mathematically sound and physically satisfactory theory by precizing some commonly used, although obviously ill-defined notions in the traditional quantum field theory. We will also proceed according to the above principles.

In our opinion, other approaches which could be used alternatively with that of Glimm and Jaffe are also worth consideration. The theory described below is very similar to the widely used theories involving boxes but, as will be clear, its physical content is completely different. We shall construct, at first, an approximate theory which will be valid for arbitrary dimensional space-time. The simple, intuitive approximation which we are going to introduce will already imply in what the sense the approximate theory has to converge.

2. *Creation and Destruction Operators of Particles with Approximated Momenta*

For simplicity, we shall consider only the case of a real scalar field. Our starting-point is to write down the purely formal expression

$$a^*(\vec{p}) = \int d^3 k \delta^{(3)}(\vec{p} - \vec{k})\, a^*(\vec{k}) \tag{2.1}$$

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where

$$a^*(\vec{p}) = \begin{cases} a^\dagger(\vec{p}) \\ a(\vec{p}) \end{cases}$$

denotes both traditionally defined creation and destruction operators of a particle with given momentum $\vec{p}$. Within the quantum mechanics we are acquainted with the following (also formal) representation of Dirac's $\delta^{(3)}$ symbol:

$$\delta^{(3)}(\vec{p}-\vec{k}) = \sum_{n=0}^{\infty} u_n(\vec{k})\, u_n{}^*(\vec{p}) \tag{2.2}$$

where $u_n(\vec{k})$ are eigenfunctions of an observable and n denotes a triplet (n_1, n_2, n_3) of integral numbers. Substituting the expansion (2.2) into (2.1) we get

$$a^*(\vec{p}) = \sum_{u=0}^{\infty} u_n{}^*(\vec{p})\, a^*(n) \tag{2.3}$$

where $u_n{}^* = u_n{}^*$ when standing next to $a^\dagger(n)$ and $u_n{}^* = u_n$ when standing next to $a(n)$, $u_n{}^*(\vec{p}) = u^*_{n_1}(p_1)\, u^*_{n_2}(p_2)\, u^*_{n_3}(p_3)$ and

$$a^*(n) = \int d^3k\, u_n{}^*(\vec{k})\, a^*(\vec{k}) \tag{2.4}$$

is a formal expression for operators which satisfy canonical commutation relations with the Kronecker symbol. Now, the possibility of some approximate calculations becomes evident. Namely, we can replace $\delta^{(3)}$ by a truncated series

$$\delta^{(3)}(\vec{p}-\vec{k}) \to \delta^{(3)}_N(\vec{p},\vec{k}) = \sum_{n=0}^{N} u_n(\vec{p})\, u_n{}^*(\vec{k}) \tag{2.5}$$

where N denotes a triplet (N_1, N_2, N_3) of finite integral numbers and we get

$$a^*(\vec{p}) \to a_N{}^*(\vec{p}) = \sum_{n=0}^{N} u_n{}^*(\vec{p})\, a^*(n) \tag{2.6}$$

which are operators on the Hilbert space of states if $u_n(\vec{p}) \in L_2$ (are square integrable. We choose as $u_{n_i}(p_i)$ the Hermite-Tschebyshev functions

$$u_{n_i}(p_i) = \frac{1}{\sqrt{(2^{n_i} n_i!\sqrt{\pi})}} \exp(-p_i{}^2/2)\, H_{n_i}(p_i) \tag{2.7}$$

[$H_{n_i}(p_i)$ are Hermite-polynomials] which span both L_2 and the Schwartz space S. The formula (7) is written in natural units $c = \hbar = l = 1$, where l is a constant with the dimension of length (Rayski, 1972).

The truncation (5) means that instead of $\delta^{(3)}$ which is a formal eigenfunction of the momentum operator some functions concentrated around $\vec{p} = \vec{k}$ from the Hilbert space are to be used. This enables us to attach to the operators $a_N{}^*(\vec{p})$ the physical interpretation of creation and destruction operators of a particle with an approximated momentum. When on builds up the theory, starting with creation and destruction operators of particles with sharply given momenta, then at first, the possibility of a comparison with physical experiments (i.e. localised in space and time), due to Heisenberg's uncertainty relations, is excluded. In the theory involving, for example, at the beginning, the $a_N^*(\vec{p})$-operators, it is not the case. We shall call the theory with $a_N^*(\vec{p})$-operators our approximate theory. Let us also mention that this approximate theory is not Lorentz-invariant, but this is not a disqualifying feature if one accepts Dirac's philosophy as we do (Dirac, 1970). It is seen that one cannot put on equal footing our approximate theory and its limit case at $N \to \infty$.

Having introduced the $a_N^*(\vec{p})$-operators we can write

$$\phi_N(\vec{x}) = \frac{1}{\sqrt{2(2\pi)^3}} \sum_{n=0}^{N} \int \frac{d^3p}{\sqrt{[\omega(\vec{p})]}} [u_n(\vec{p})\, a(n) \exp(i\vec{p}\vec{x}) + u_n^*(\vec{p})\, a^\dagger(n) \exp(-i\vec{p}\vec{x})] \tag{2.8}$$

with $\omega(\vec{p}) = \sqrt{(\vec{p}^2 + m^2)}$ and define the operator

$$H_N = H_{ON} + H_{IN} \tag{2.9}$$

where, in analogy to the hamiltonian of the free field

$$H_{ON} = \int d^3p\, \omega(\vec{p})\, a_N^\dagger(\vec{p})\, a_N(\vec{p}) \tag{2.10}$$

and H_{IN} is also an expression in which traditional $a^*(\vec{p})$-operators are to be replaced by the $a_N^*(\vec{p})$-operators. As the next step we define

$$\phi_N(\vec{x}, t) = \exp(-iH_N t)\, \phi_N(\vec{x}) \exp(iH_N t) \tag{2.11}$$

Thus, this time-dependent operator is to be understood as an approximate solution of field equations. Finally, we will evaluate the following scalar products:

$$W_k^N(\vec{x}_1 t_1, \ldots, \vec{x}_k t_k) = [\psi_{ON}, \phi_N(\vec{x}_1 t_1), \ldots, \phi_N(\vec{x}_k t_k)\, \psi_{ON}] \tag{2.12}$$

with ψ_{ON} denoting the ground eigenstate of the H_N-operator. We wrote (2.12) in analogy to Wightman's distributions and we shall call them approximate distributions. The $\phi_N(\vec{x}t)$-operators are not depending upon the $\vec{x}$-variable within generalised functions but within ordinary functions. Consequently, our approximate distributions (2.12) will be nothing else but just ordinary functions (regular distributions).

Now, the programme is to investigate the convergence of sequences of our approximate distributions (2.12) at $N \to \infty$ for each fixed k. If corresponding limits were to be shown to exist in the distributional sense and the limit distributions would manifest all the characteristics of Wightmann's distributions then, referring to Wightmann's reconstruction theorem, we could state that a plausible approximation in the quantum field theory has been constructed. We can write (2.12) as

$$\sum_{n_1}^{N} \cdots \sum_{n_k}^{N} \frac{1}{\{\sqrt{[2(2\pi)^3]}\}^k} \int \frac{d^3_{p_1}, \ldots, d^3_{p_k}}{\sqrt{[\omega(\vec{p}_1), \ldots, \omega(\vec{p}_k)]}} u^*_{n_1}(\vec{p}_1), \ldots, u^*_{n_k}(\vec{p}_k) \times$$
$$[\psi_{ON}, a^{*N}(n_1 t_1), \ldots, a^{*N}(n_k t_k)\psi_{ON}] \exp(i \sum_j \varepsilon_j \vec{p}_j \vec{x}_j) \tag{2.12a}$$

where ε_j is 1 or -1 and

$$a^{*N}(nt) = \exp(-iH_N t)\, a^*(n) \exp(iH_N t) \tag{2.13}$$

and we can see that our approximate distributions are linear combinations of momentum integrals—with weight $[\exp(\pm i\vec{p}_j \vec{x}_j)]\,[\omega(\vec{p}_i)]^{-1/2}$—where integrands are of the type

$$\sum_{n_1}^{N} \cdots \sum_{n_k}^{N} u_{n_1}^*(\vec{p}_1), \ldots, u_{n_k}^*(\vec{p}_k)\,[\psi_{ON}, a^{*N}(n_1 t_1), \ldots, a^{*N}(n_k t_k)\psi_{ON}] \tag{2.12b}$$

The convergence of the sequences of approximate distributions (2.12) will be provided by the existence of limits of all the expressions (2.12b). Such an expression has been discussed by Mikusiński (1968), who gave the sufficient condition for the existence of its limit in the distributional sense. This condition means that there must exist such finite numbers K and L that at fixed times $t_1, \ldots, t$ and for all $n_1, \ldots, n$ the following inequality must be satisfied:

$$\frac{(\psi_{ON}, a^{*N}(n_1 t_1), \ldots, a^{*N}(n_k t_k)\,\psi_{ON})}{(1 + n_1{}^L), \ldots, (1 + n_k{}^L)} < K \tag{2.14}$$

In this condition (which we shall refer as the Mikusiński condition) the $a^*(n)$-operators are involved and they are known, and do not depend, for example, on the H_N-choice. Since the ψ_{ON}-state is determined by the H_N-operator, the problem whether the Mikusiński condition holds or does not hold is dependent only on the way H_N is expressed by the $a^*(n)$-operators. Hence, the existence of the limit case depends on the model, i.e. on the particular shape of the H_N-operator.

3. *The Free Field*

The programme introduced in the foregoing section should be executed at first for a free field where the shape of Wightmann's distributions is known. We define the free field case by

$$H_N = H_{ON} \tag{3.1}$$

and ϕ_{ON} is to be understood as the ground eigenstate of the H_{ON}-operator. The existing free field theory will be called here 'the exact theory'. By showing that some numerical results obtained in low orders of our approximation (i.e. for small N) do not differ much from the results of the exact theory it will be made plausible that our approximation is useful for some numerical computations in a theory with interaction.

Let us focus attention on the H_{ON}-operator given by (2.10). When introducing the hermitian matrix M:

$$M_{nn'} = \int d^3p\, \omega(\vec{p})\, u_n{}^*(\vec{p})\, u_{n'}(\vec{p}) \tag{3.2}$$

it can be written as follows:

$$H_{ON} = \sum_{n=0}^{N} \sum_{n=0}^{N} M_{nn'} a^{\dagger}(n) a(n') \tag{3.3}$$

or

$$H_{ON} = \sum_{n=0}^{N} \lambda_n \tilde{a}^{\dagger}(n)\, \tilde{a}(n) \tag{3.3a}$$

where $\lambda_0, \ldots, \lambda_N \geqslant 0$ are eigenvalues of M (which is positive definite) and $\tilde{a}^*(n)$ are obtained from $a^*(n)$ within a unitary transformation which was chosen to diagonalise M. ϕ_{ON} must obey $\tilde{a}^{\dagger}(n)\, \tilde{a}(n)\, \phi_{ON} = 0$ for each n and, consequently, $a(n)\, \phi_{ON} = 0$. In view of the irreducibility of the representation we get

$$\psi_{ON} = \psi_0 \tag{3.4}$$

i.e. ψ_{ON} is the same as the vacuum in each order N.

On the standard way we can express the $a^{*N}(nt)$-operators (2.13) by $a^*(n)$:

$$a^{\dagger N}(nt) = \sum_{n'=0}^{N} [\exp(iMt)]_{n'n} a^{\dagger}(n'), \qquad a^{N}(nt) = \sum_{n'=0}^{N} [\exp(-iMt)]_{nn'}\, a(n') \tag{3.5}$$

The Mikusiński condition (2.14) is to be verified for all k separately. For $k = 2$ the only contribution to $[\psi_0, a^{*N}(n_1 t_1) a^{*N}(n_2 t_2) \psi_0]$ will be $(\psi_0, a^N(n_1 t_1) a^{\dagger N}(n_2 t_2) \psi_0)$, as is seen from (3.5), and we have

$$W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2) = \frac{1}{2(2\pi)^3} \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} \int \frac{d^3 p_1\, d^3 p_2}{\sqrt{[\omega(\vec{p}_1)\, \omega(\vec{p}_2)]}} \exp[i(\vec{p}_1 \vec{x}_1 - \vec{p}_2 \vec{x}_2)] \{\exp[-iM(t_1 - t_2)]\}_{n_1 n_2} u_{n_1}(\vec{p}_1)\, u^*_{n_2}(\vec{p}_2) \tag{3.6}$$

As one can see, in the $k = 2$ case the Mikusiński condition holds because of

$$|(\{\exp[-iM(t_1 - t_2)]\})_{n_1 n_2}| < \infty \tag{3.7}$$

For odd k the limit also exists because we have

$$W_k{}^N(\vec{x}_1 t_1, \ldots, \vec{x}_k t_k) = 0 \quad \text{for odd } k \tag{3.8}$$

and for even k, $k > 2$ one can easily show that the Mikusiński condition also holds, provided it holds for $k = 2$.

Since the limits of sequences of our approximated distributions exist for all k, we can try to find them. We have immediately

$$\lim_{n \to \infty} W_k{}^N(\vec{x}_1 t_1, \ldots, \vec{x}_k t_k) = 0 \quad \text{for odd } k \tag{3.8a}$$

Furthermore, it will do to find the limit of $W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2)$ at $N \to \infty$. In order to do so we introduce

$$I_N(\vec{p}_1, \vec{p}_2) = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} u_{n_1}(\vec{p}_1) \{\exp[-iM(t_1 - t_2)]\}_{n_1 n_2} u^*_{n_2}(\vec{p}_2) \tag{3.9}$$

and the problem is to find

$$\lim_{N\to\infty} W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2)$$

$$= \lim_{N\to\infty} \left(\frac{1}{2(2\pi)^3} \int \frac{d^3p_1\, d^3p_2}{\sqrt{[\omega(\vec{p}_1)\,\omega(\vec{p}_2)]}} \exp(i\vec{p}_1\vec{x}_1 - \vec{p}_2\vec{x}_2)\, I_N(\vec{p}_1,\vec{p}_2) \right) \quad (3.10)$$

where the convergence has to be understood in the distributional sense. The introduced $I_N(\vec{p}_1,\vec{p}_2)$ can be written as follows:

$$I_N(\vec{p}_1,\vec{p}_2) = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} \sum_{k=0}^{\infty} \frac{[i(t_2-t_1)]^k}{k!} u_{n_1}(\vec{p}_1)\,(M^k)_{n_1 n_2}\, u^*_{n_2}(\vec{p}_2)$$

$$= \sum_{k=0}^{\infty} \frac{[i(t_2-t_1)]^k}{k!} \sum_{n_1=0}^{N} \sum_{\nu_2=0}^{N} \sum_{\nu_1=0}^{N} \cdots \sum_{\nu_{k-1}=0}^{N} u_{n_1}(\vec{p}_1)\, M_{n_1\nu_2} M_{\nu_1\nu_2}, \ldots, M_{\nu_{k-1}n_2}\, u^*_{n_2}(\vec{p}_2)$$

$$= \sum_{k=0}^{\infty} \frac{[i(t_2-t_1)]^k}{k!} \int d^3q_1\, \omega(\vec{q}_1) \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} \sum_{\nu_1=0}^{N} \cdots \sum_{\nu_{k-1}=0}^{N} \times \cdots$$

$$u_{n_1}(\vec{p}_1)\, u^*_{n_1}(\vec{q}_1)\, u^*_{\nu_1}(\vec{q}_1)\, M_{\nu_1\nu_2}\, M_{\nu_{k-1}n_2}\, u^*_{n_2}(\vec{p}_2)$$

and denoting

$$K_N(\vec{p}_1,\vec{q}_1) = \sum_{n_1=0}^{N} u_{n_1}(\vec{p}_1)\, u^*_{\nu_1}(\vec{q}_1)$$

$$K_N(\vec{q}_1,\vec{q}_2) = \sum_{\nu_1=0}^{N} u_{\nu_1}(\vec{q}_1)\, u^*_{v_1}(\vec{q}_2)$$

. . . .

we get

$$I_N(\vec{p}_1,\vec{p}_2) = \sum_{k=0}^{\infty} \frac{[i(t_2-t_1)]^k}{k!} \int d^3q_1, \cdots, d^3q_k\, \omega(\vec{q}_1), \cdots,$$

$$\omega(\vec{q}_k)\, K_N(\vec{p}_1,\vec{q}_1), \cdots, K_N(\vec{q}_k,\vec{p}_2) \quad (3.11)$$

The sequence of regular distributions $W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2)$ converges, whence we can attach to $I_N(\vec{p}_1,\vec{p}_2)$ an interpretation of a convergent subsequence of the following sequence:

$$I_{N1}, \cdots, N_{k+1}(\vec{p}_1,\vec{p}_2)$$

$$= \sum_{k=0}^{\infty} \frac{[i(t_2-t_1)]^k}{k!} \int d^3q_1, \cdots, d^3q_k\, \omega(\vec{q}_1), \cdots, \omega(\vec{q}_k)\, K_{N_1}(\vec{p}_1,\vec{q}_1), \cdots,$$

$$K_{N_{k-1}}(\vec{q}_k,\vec{p}_2) \quad (3.12)$$

and write

$$\lim_{N\to\infty} I_N(\vec{p}_1,\vec{p}_2) = \lim_{N_1,\ldots,N_{k+1}\to\infty} I_{N_1,\ldots,N_{k+1}}(\vec{p}_1,\vec{p}_2) \quad (3.13)$$

Thus, finding, for example, the limit at $N_1 \to \infty$, becomes the same as evaluating

$$\lim_{N_1\to\infty} \int d^3q_1\, K_{N_1}(\vec{p}_1,\vec{q}_1) f(q_1) = \lim_{N_1\to\infty} \int d^3q_1 \sum_{n=0}^{N_1} u_n(\vec{p}_1)\, u_n{}^*(\vec{q}_1)\, f(\vec{q}_1)$$

where $f(\vec{q}_1) = \omega(\vec{q}_1) K_{N_2}(\vec{q}_1, \vec{q}_2)|_{q_2=\text{const.}}$ For $f(\vec{q}_1) \in S(\mathbb{R}^3)$ the last limit is equal to $f(\vec{p}_1)$. In other words, we have

$$\lim_{N_1 \to \infty} K_{N_1}(\vec{q}_1, \vec{p}_1) = \delta^{(3)}(\vec{p}_1 - \vec{q}_1)$$

and at $N_1, \ldots, N_{k+1} \to \infty$ we get

$$\lim_{N_1 \to \infty} I_N(\vec{p}_1, \vec{p}_2) = \exp[-i\omega(\vec{p}_1)(t_1 - t_2)]\, \delta^{(3)}(\vec{p}_1 - \vec{p}_2) \tag{3.14}$$

The sequence $W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2)$ has to converge in the distributional sense and the problem (3.11) reads finding

$$\lim_{N \to \infty} \int d^3x_1\, d^3x_2\, g_1{}^*(\vec{x}_1)\, g_2(\vec{x}_2)\, W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2)$$

where $g_1(\vec{x}_1)$ and $g_2(\vec{x}_2)$ are some test functions from $S(\mathbb{R}^3)$. Making use of the result (3.14) we get

$$\lim_{N \to \infty} \int d^3x_1\, d^3x_2\, g_1{}^*(\vec{x}_1)\, g_2(\vec{x}_2)\, W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2)$$
$$= \frac{1}{2} \int \frac{d^3p}{\omega(\vec{p})} \tilde{g}_1{}^*(\vec{p}_1)\, \tilde{g}_2(\vec{p}_2) \times \exp[-i\omega(\vec{p})\,(t_1 - t_2)] \tag{3.15}$$

where $\tilde{g}$ denotes the Fourier-transform of g. In other words, we have in the distributional sense

$$\lim_{N \to \infty} W_2{}^N(\vec{x}_1 t_1, \vec{x}_1 t_2) = \frac{1}{2(2\pi)^3} \int \frac{d^3p}{\omega(\vec{p})} \exp\{i[\vec{p}(\vec{x}_1 - \vec{x}_2) - \omega(\vec{p})\,(t_1 - t_2)]\} \tag{3.16}$$

which is just the two-point Wightmann's distribution representing the exact theory. Thus, we have shown that the limit case of our approximate theory is nothing else but, as we could expect, the well-known free field theory.

Let us discuss how much some numerical result of our approximate theory and the exact theory differ one from the other. In order to do that we should compare the integrals

$$\int d_3x_1\, d^3x_2\, g_1{}^*(\vec{x}_1)\, g_2(\vec{x}_2)\, W_2{}^N(\vec{x}_1 t_1, \vec{x}_2 t_2) \quad \text{for } N = 0, 1, 2, \ldots$$

with the formula (3.15). Let us consider the $t_1 = t_2$ case only, while $g_1(\vec{x}_1)$ and $g_2(\vec{x}_2)$ can be chosen so that $g_1 = g_2 = g$ and

$$\tilde{g}(\vec{p}) = \sum_{n=0}^{K} a_n\, u_n(\vec{p}) \sqrt{[\omega(\vec{p})]} \tag{3.17}$$

with finite K and a_n being some numbers. For the exact theory we get, when substituting (3.17) into (3.15),

$$\frac{1}{2} \int \frac{d^3p}{\omega(\vec{p})} |\widetilde{g(\vec{p})}|^2 = \frac{1}{2} \int d^3p \left| \sum_{n=0}^{K} a_n\, u_n(\vec{p}) \right|^2 = \frac{1}{2} \sum_{n=0}^{K} |a_n|^2 \tag{3.18}$$

and for the approximate theory, when coming back to the formula (3.6) we get for $N = 0, 1, 2, \ldots$

$$\frac{1}{2}\int \frac{d^3p_1\, d^3p_2}{\sqrt{[\omega(\vec{p}_1)\, \omega(\vec{p}_2)]}}\, \tilde{g}^*(\vec{p}_1)\, \tilde{g}(\vec{p}_2) \sum_{n=0}^{N} u_n(\vec{p}_1)\, u_n^*(\vec{p}_2)$$

$$= \frac{1}{2}\sum_{n=0}^{N}\sum_{i,j=0}^{K} a_i^*\, a_j \int d^3p_1\, d^3p_2\, u_i^*(\vec{p}_1)\, u_j(\vec{p}_2)\, u_n(\vec{p}_1)\, u_n^*(\vec{p}_2)$$

$$= \frac{1}{2}\sum_{n=0}^{N} |a_n|^2 \tag{3.19}$$

When comparing the results of (3.18) and (3.19) it is seen that in order to obtain identical results it would be sufficient to take the order of approximation $N = K$. However, this conclusion is due to test functions of the form (3.17) which for finite K do not span the Schwarz space. Summarising, in the free field case and for certain (extensible when taking $N > K$) classes of equal-time states the approximate theory and the exact theory should yield identical results.

In the case of interaction a theory corresponding to the known free field theory (which we called 'the exact theory') has not been yet formulated, and we could only compare numerical results of an approximate theory with interaction with those of its limit case. However, it would mean that this limit case is assumed to be just a quantum field theory with interaction. We think such an assumption to be plausible. Thus, the given construction involving the Mikusiński condition implies one of the possibilities of how to attempt the proof of existence of non-trivial models.

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